

## Binomial Theorem

1. Find the term independent of  $x$ ,  $x \neq 0$ , in the expansion of  $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$ .
2. If the term free from  $x$  in the expansion of  $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$  is 405, find the value of  $k$ .
3. Find the coefficient of  $x$  in the expansion of  $(1 - 3x + 7x^3)(1 - x)^{16}$ .
4. Find the term independent of  $x$  in the expansion of,  $\left(3x - \frac{2}{x^2}\right)^{15}$ .
5. Find the middle term (terms) in the expansion of
  - (i)  $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$
  - (ii)  $\left(3x - \frac{x^3}{6}\right)^9$
6. Find the coefficient of  $x^{15}$  in the expansion of  $(x - x^2)^{20}$ .
7. Find the coefficient of  $\frac{1}{x^{17}}$  in the expansion of  $\left(x^4 - \frac{1}{x^3}\right)^{15}$ .
8. Find the sixth term of the expansion  $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$ , if the binomial coefficient of the third term from the end is 45.  
[Hint: Binomial coefficient of third term from the end = Binomial coefficient of third term from beginning =  ${}^nC_2$ .]
9. Find the value of  $r$ , if the coefficients of  $(2r + 4)^{\text{th}}$  and  $(r - 2)^{\text{th}}$  terms in the expansion of  $(1 + x)^{18}$  are equal.
10. If the coefficient of second, third and fourth terms in the expansion of  $(1 + x)^{2n}$  are in A.P. Show that  $2n^2 - 9n + 7 = 0$ .
11. Find the coefficient of  $x^4$  in the expansion of  $(1 + x + x^2 + x^3)^{11}$ .

12. If  $p$  is a real number and if the middle term in the expansion of  $\left(\frac{p}{2} + 2\right)^8$  is 1120, find  $p$ .

13. Show that the middle term in the expansion of  $\left(x - \frac{1}{x}\right)^{2n}$  is  $\frac{1 \times 3 \times 5 \times \dots \times (2n-1)}{\underline{n}} \times (-2)^n$ .

14. Find  $n$  in the binomial  $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$  if the ratio of 7<sup>th</sup> term from the beginning to the 7<sup>th</sup> term from the end is  $\frac{1}{6}$ .

15. In the expansion of  $(x + a)^n$  if the sum of odd terms is denoted by  $O$  and the sum of even term by  $E$ .

Then prove that

(i)  $O^2 - E^2 = (x^2 - a^2)^n$                       (ii)  $4OE = (x + a)^{2n} - (x - a)^{2n}$

16. If  $x^p$  occurs in the expansion of  $\left(x^2 + \frac{1}{x}\right)^{2n}$ , prove that its coefficient is

$$\frac{\underline{2n}}{\frac{4n-p}{3} \frac{2n+p}{3}}$$

17. Find the term independent of  $x$  in the expansion of  $(1 + x + 2x^2) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ .

## Objective Type Questions

Choose the correct answer from the given options in each of the Exercises 18 to 24 (M.C.Q.).

18. The total number of terms in the expansion of  $(x + a)^{100} + (x - a)^{100}$  after simplification is  
 (A) 50 (B) 202 (C) 51 (D) none of these
19. Given the integers  $r > 1$ ,  $n > 2$ , and coefficients of  $(3r)^{\text{th}}$  and  $(r + 2)^{\text{th}}$  terms in the binomial expansion of  $(1 + x)^{2n}$  are equal, then  
 (A)  $n = 2r$  (B)  $n = 3r$  (C)  $n = 2r + 1$  (D) none of these
20. The two successive terms in the expansion of  $(1 + x)^{24}$  whose coefficients are in the ratio 1:4 are  
 (A) 3<sup>rd</sup> and 4<sup>th</sup> (B) 4<sup>th</sup> and 5<sup>th</sup> (C) 5<sup>th</sup> and 6<sup>th</sup> (D) 6<sup>th</sup> and 7<sup>th</sup>

[Hint:  $\frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4} \Rightarrow \frac{r+1}{24-r} = \frac{1}{4} \Rightarrow 4r+4 = 24-4 \Rightarrow \boxed{r=4}$ ]

21. The coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n}$  and  $(1 + x)^{2n-1}$  are in the ratio.  
 (A) 1:2 (B) 1:3 (C) 3:1 (D) 2:1

[Hint:  ${}^{2n}C_n : {}^{2n-1}C_n$ ]

22. If the coefficients of 2<sup>nd</sup>, 3<sup>rd</sup> and the 4<sup>th</sup> terms in the expansion of  $(1 + x)^n$  are in A.P., then value of  $n$  is  
 (A) 2 (B) 7 (C) 11 (D) 14

[Hint:  $2 {}^nC_2 = {}^nC_1 + {}^nC_3 \Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 2 \text{ or } 7$ ]

23. If A and B are coefficient of  $x^n$  in the expansions of  $(1 + x)^{2n}$  and  $(1 + x)^{2n-1}$  respectively, then  $\frac{A}{B}$  equals

- (A) 1 (B) 2 (C)  $\frac{1}{2}$  (D)  $\frac{1}{n}$

[Hint:  $\frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = 2$ ]

24. If the middle term of  $\left(\frac{1}{x} + x \sin x\right)^{10}$  is equal to  $7\frac{7}{8}$ , then value of  $x$  is

$$(A) \ 2n\pi + \frac{\pi}{6} \quad (B) \ n\pi + \frac{\pi}{6} \quad (C) \ n\pi + (-1)^n \frac{\pi}{6} \quad (D) \ n\pi + (-1)^n \frac{\pi}{3}$$

$$[\text{Hint: } T_6 = {}^{10}C_5 \frac{1}{x^5} \cdot x^5 \sin^5 x = \frac{63}{8} \Rightarrow \sin^5 x = \frac{1}{2^5} \Rightarrow \sin = \frac{1}{2}$$

$$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6}]$$